Attachment “6"

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Relation between Monopole Mass and Primary Monopole Flux*

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We give quantitative results concerning the simple idea that the probability of observing a monopole at Earth with $\beta \simeq 0.5$ decreases as the monopole mass decreases, for masses $\lesssim 10^4$ Gev/c^2: The effective cross section of Earth is roughly the geometric cross section times the ratio of the monopole mass to 8000 GeV/c^2.

Recently Price *et al.*,[1] reported a cosmic-ray event which they interpret as a magnetic monopole[2,3] of charge 137e, and mass $\gtrsim 600 M_p$ (M_p is the proton mass). It was observed near Sioux City, Iowa (latitude ~42.5°N) to be going generally downward with a velocity $\simeq 0.5c$. The purpose of the present Comment is to make quantitative a rather obvious point: The lighter the monopole, the less likely it is to be observed anywhere on its trajectory with such a low velocity.[4] The reason is that the potential energy at the surface of Earth of a monopole of magnetic charge $g = 137e$ in Earth's dipole field is roughly 8000 GeV, and a monopole of mass much less than this must nearly always be traveling at close to the speed of light. Only monopoles of mass $\gtrsim 10^4$ GeV/c^2 have much chance of retaining a small velocity in Earth's magnetic field.

For the purposes of this work, we assume that the alleged monopole came from outer space as a primary cosmic ray. Wilson[5] and Hungerford[4] have shown that, for kinematic reasons, the monopole could not have been produced in the atmosphere by a conventional cosmic-ray primary. We do not consider the possibility that the monopole migrated to the surface of Earth from the interior, and was then accelerated along a field line. One might expect a cosmic monopole to have been accelerated to very high energies by galactic magnetic fields, in which case the observed event is *a priori* an unlikely candidate for a monopole. However, it might have come from the Sun at low energies in which case its energy change in the solar magnetic field might well be comparable to its potential energy in Earth's field.

We have studied the orbits of a monopole in Earth's field by a combination of computer-generated orbits and approximate analysis. Let us begin with the analysis. Earth's magnetic field is approximated by a centered dipole of magnetic moment $B_0 a^3$, where a is the radius of Earth (6371 km) and $B_0 = 0.31$ G. In units which are natural for the problem (length measured in units of a, velocity in units of c, and mass or energy in units of the monopole mass M), the equations of motion are

$$d\vec{p}/dt = -K\nabla[(\cos\theta)/r^2], \tag{1}$$

where $\vec{p} = \gamma\vec{\beta}$ is the momentum $[\gamma = (1-\beta^2)^{-1/2}]$, and

$$K = gB_0 a/Mc^2 = 8641 M_p/M. \tag{2}$$

The corresponding Hamiltonian is

$$H = \gamma + K(\cos\theta)/r^2, \tag{3}$$

and it is conserved, as is the azimuthal momentum $P_\varphi = \gamma r^2 \sin^2\theta\,\dot{\varphi}$.

The orbital motion can be reduced to quadratures for nonrelativistic motion. Such an analysis will be relevant in the near vicinity of the place where the cosmic-ray event was observed, where the motion is nearly nonrelativistic.

The nonrelativistic equation for θ reads

$$\frac{d(r^2\dot{\theta})}{dt} = \frac{K\sin\theta}{r^2} + \frac{P_\varphi^{\,2}\cos\theta}{r^2\sin^3\theta}, \tag{4}$$

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and it can be integrated to yield

$$r^2\dot{\theta} = (C_1 - 2K\cos\theta - P_\varphi^2/\sin^2\theta)^{1/2}, \qquad (5)$$

where C_1 is a constant. This, substituted into the nonrelativistic version of (3), yields

$$\dot{r}^2 = 2(H-1) - C_1/r^2, \qquad (6)$$

which is the same as for free-particle motion ($K = 0$). Then (5) and (6) combine to give

$$\int d\theta (C_1 - 2K\cos\theta - P_\varphi^2/\sin^2\theta)^{1/2} = \int dr\, r^{-1}[2r^2(H-1) - C_1]^{-1/2}$$
$$= C_1^{-1/2}\cos^{-1}(r_0/r) + \text{const}, \quad r_0^2 = \tfrac{1}{2}C_1(H-1)^{-1}. \qquad (7)$$

In the limit of free-particle motion, r_0 would be the impact parameter; in any event, it is a turning point ($\dot{r} = 0$).

First consider the case $K \gg 1$; that is, the particle rest-mass energy is much less than its potential energy in Earth's field. We shall consider only the case where this potential energy is positive at the point of striking Earth (a south-seeking monopole); otherwise, the *total* energy H is negative and the observed event could not have been a primary from outer space.

Let θ_E be the colatitude of the monopole as it strikes Earth; by the remarks in the last paragraph, $\theta_E < \pi/2$. For simplicity's sake, we consider only the case $P_\varphi = 0$, although our conclusions are unchanged in the more general case. It is evident from (5) that $C_1 > 2K\cos\theta_E$. Furthermore, since the observed event has $\beta \simeq 0.5$ at Earth ($r = 1$),[6] the left-hand side of (5) is bounded at Earth by $0.5\sin\alpha$, where α is the angle the velocity vector makes with the local zenith. This yields the inequalities

$$2K\cos\theta_E \le C_1 \le 2K\cos\theta_E + \tfrac{1}{4}\sin^2\alpha. \qquad (8)$$

For large K, C_1 can vary in only a small range around its average value. Similarly, $\dot{r}^2$ at $r = 1$ is bounded by $\frac{1}{4}\cos^2\alpha$, and (6) yields inequalities on H:

$$1 + K\cos\theta_E \le H \le 1 + \tfrac{1}{8} + K\cos\theta_E. \qquad (9)$$

The fractional change in H is $(8K\cos\theta_E)^{-1}$, or about $\frac{1}{80}$ for a monopole of $M = 600M_p$ over Sioux City. This is a factor of 10 smaller than the allowed fractional change in H for a very heavy ($K \ll \frac{1}{8}$) monopole.

We may use the bound (9) to conclude that, for $K \ll 1$, there is a turning point in the r motion very near where the monopole has $\beta \simeq 0.5$. The turning point occurs when the right-hand side of (6) vanishes; using the upper limit for H and $\alpha = \pi$ yields a turning point at [see (7)]

$$r = r_0 = [K\cos\theta_E/(\tfrac{1}{8} + K\cos\theta_E)]^{1/2}, \qquad (10)$$

or about 60 km away from the observation point (taken to be $r = 1$) for $M = 1000M_p$. Furthermore, in the same case (6) predicts $\dot{r} = 1$ about 160 km above the observation point [of course, this is not really accurate because the nonrelativistic approximation breaks down; it is more accurate to say that a particle has $\gamma \simeq 1.5$ where (6) predicts $\dot{r} = 1$].[7] However, the picture is clear: Metaphorically, the probability of observing a light ($K \gg 1$) monopole with $\beta \simeq 0.5$ is akin to the probability of observing a tennis ball at rest (as it strikes a racket or the ground) during the course of a point played in a tennis game.

If r_0 changes by as little as $\pm(4K\cos\theta_E)^{-1}$ either the turning point will move above Earth so that the monopole will not be detected, or it moves down enough so that the velocity at detection will be nearly the speed of light. Thus the constants of the motion H and C_1 must be very carefully adjusted if the observed event is to be a light monopole. If H or C_1 changes by about ± 1, r_0 will change as described above. The significance of H is immediately apparent, but the significance of C_1 remains to be discussed.

For the relativistic case, and for motion in the azimuthal plane, the exact analog of Eq. (4) can be written in the following somewhat artificial way:

$$\frac{1}{2}\frac{d(\gamma^{1/2}r^2\dot{\theta})^2}{d\theta} = K\sin\theta - \frac{1}{2}(r^2\dot{\theta})^2\frac{d\gamma}{d\theta}. \qquad (11)$$

It is a tolerable approximation to drop the second term on the right-hand side of (11) for these reasons: Near Earth, $d\gamma/d\theta$ is of order $K\sin\theta$, but $\frac{1}{2}(r^2\dot{\theta})^2$ is $\sim\frac{1}{8}$. Far from Earth, $d\gamma/d\theta$ vanishes like r^{-1}, while $r^2 \to b$, the impact parameter. It follows that the nonrelativistic integral (5) can be replaced by[7]

$$\gamma^{1/2}r^2\dot{\theta} = (C_1 - 2K\cos\theta)^{1/2}. \qquad (12)$$

The approximate constancy of C_1 as defined by (12) has been checked with the computer. For asymptotically large r, $\gamma \to H \simeq 1 + K\cos\theta_E$, $r^2\dot{\theta} \to b$, and $\theta \to \theta_0$, where θ_0 is the polar angle of the

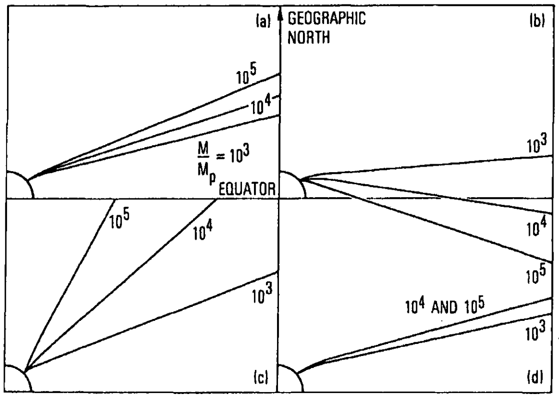

FIG. 1. Monopole trajectories for various monopole masses M, projected onto an azimuthal plane and displayed in geographical coordinates. Four different orientations of the initial velocity vector β are used: (a) $\beta \parallel B$; (b) rotated 45° south from (a); (c) rotated 45° north from (a); (d) rotated 45° east from (a).

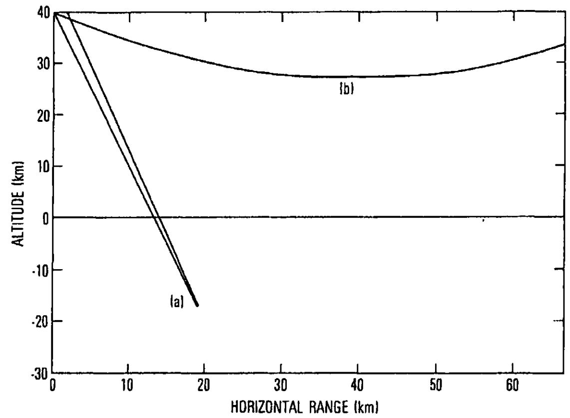

FIG. 2. Turning points for forward-going orbits of a south-seeking monopole, beginning from the point of observation at altitude 40 km. The mass is $1000M_p$, and cases (a) and (b) of Fig. 1 are shown.

impact-parameter vector $\vec{b}$, and we conclude

$$C_1 \simeq 2K\cos\theta_0 + b^2(1+K\cos\theta_E). \tag{13}$$

It is evident that a very small change in impact parameter of $O((2bK\cos\theta_E)^{-1})$ will produce unit change in C_1. It is useful to interpret this change in b in terms of an equivalent cross section, defined as

$$\sigma = 2\pi \int_R b\,db, \tag{14}$$

where R is the accessible region of impact parameters corresponding to the monopole as observed at Earth. (Earth's geometric cross section is π, or πa^2 in more conventional units.) Evidently, $\sigma \simeq \pi(1+K\cos\theta_E)^{-1}$, from the remarks below Eq. (13).

We conclude that for $K \ll 1$, a monopole corresponding to the observations reported in Ref. 1 has a logarithmic energy window $\Delta H/H$ of $\sim[8(1+K\cos\theta_E]^{-1}$, and a cross section of about $\pi a^2(1+K\cos\theta_E)^{-1}$ for striking Earth.

The computer-calculated orbits which support this analysis were done with a standard magnetic-field model[8] which is fitted to observations, not with the dipole field of Eq. (1). The model does not include solar-wind distortions, but these are negligible out to about 10 Earth radii. The analogs of Eq. (1) were integrated both backward and forward from the initial conditions, always with initial $\beta \simeq 0.5$, and for masses from $10^3 M_p$ to $10^5 M_p$. For a south-seeking monopole, the backward orbits are rather uninteresting; they are shown in Fig. 1 for several cases. However, the forward orbits show, as expected, turning points for the smaller masses, as shown in Fig. 2, in rough agreement with the analysis. Numerous other cases were run, e.g., including ionization loss, north-seeking monopoles, and all results are consistent with expectations.

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[1]P. B. Price, E. K. Shirk, W. Z. Osborne, and L. S. Pinsky, Phys. Rev. Lett. 35, 487 (1975). The lower-limit mass of $600M_p$ is quoted in Phys. Today 28, No. 10, 17 (1975).

[2]P. A. M. Dirac, Proc. Roy. Soc. London, Ser. A 133, 60 (1931).

[3]J. Schwinger, Phys. Rev. 173, 1536 (1968). The charge attributed to the monopole by Price *et al.* is twice the lowest value allowed by Dirac, and equal to the lowest value allowed by Schwinger.

[4]E. V. Hungerford, Phys. Rev. Lett. 35, 1303 (1975), has in effect made the same point, although he has not commented on the turning point for a positive-energy monopole. His work appeared just as we were completing our work, which goes beyond Hungerford's in having analytical formulas as well as orbits computed for a wider variety of circumstances.

[5]Lance W. Wilson, Phys. Rev. Lett. 35, 1126 (1975).

[6]The observed event had traversed some 3 g cm^{-2} of air above the balloon; if it were a monopole of charge $137e$, it would have lost $\simeq 60$ GeV by ionization losses. Thus β could have been somewhat greater than 0.5, in the absence of an atmosphere. Since 60 GeV is much less than the energy scale of ~8000 GeV, this should be a negligible effect. The computer calculations show

that our main conclusions are unaffected by adding ionization losses.

[7]Corresponding to the approximation, (12) is a more accurate version of (6), in which the left-hand side is replaced by $2[(1-\dot{r}^2)^{-1/2}-1]$, to be used in the vicinity of Earth where $C_1 - 2K\cos\theta$ is small.

[8]IAGA Commission 2, Working Group 4, J. Geophys. Res. 74, 4407 (1969).